Boolean Algebra

proposition: An expression that can be true or false

e.g. The sun will rise tomorrow.

It rained yesterday.

George Boole: An English mathematician who invented an algebra named Boolean Algebra which deals with only two values: true and false.

Comparison

Regular Algebra	Boolean Algebra
Numbers	true (1), false (0)
operations: +, −, *, /	operations: AND (•), OR (+), XOR (⊕), NOT (')
e.g. $5-2*3=-1$	true • false + true = tru $1 \cdot 0 + 1 = 1$

Boolean Variables

A variable is a "box" containing a specific value at a time

B | true — now

B | false — later

1 ≡ true 0 ≡ false

AND

$A \cdot B = C$

A	B	C
0	0	0
0	1	0
1	0	0
1	1	1

OR

$A + B = C$

A	B	C
0	0	0
0	1	1
1	0	1
1	1	1

XOR

$A \oplus B = C$

A	B	C
0	0	0
0	1	1
1	0	1
1	1	0

Addition (Keep 1 digit)

NOT

$A' = C$

A	$A' = C$
0	1
1	0

NAND

$(A \cdot B)'$

NOR

$(A + B)'$

University Admissions policy

A Student will be accepted if the Student's
1. High school GPA is 3.0 or better
and 2. SAT score is 1100 or higher
(combined math and reading)

~~A = B = C~~ $A = G \cdot S$

G GPA	S SAT	A Acceptance
3.2 G = true	1150 S = true	Yes A = true
3.5 G = true	1060 S = false	No A = false
2.8 G = false	950 S = false	A = false

What if and $\longrightarrow$ or

$$A = G + S$$

Boolean Algebra properties

properties	AND	OR
commutative	~~$(AB)C = A(BC)$~~ $AB = BA$	$A+B = B+A$
Associative	$(AB)C = A(BC)$	$(A+B)+C = A+(B+C)$
Distributive	$A(B+C) = AB+AC$	$A+(B\cdot C) = (A+B)\cdot(A$
Identity	$A\cdot 1 = A$	$A+1 = 1$
complement	$A\cdot 0 = 0$, $A\cdot A' = 0$	$A+0 = A$, and $A+A' = 1$
DeMorgan's law	$(A\cdot B)' = A'+B'$	$(A+B)' = A'\cdot B'$

A, B, and C are Boolean variables

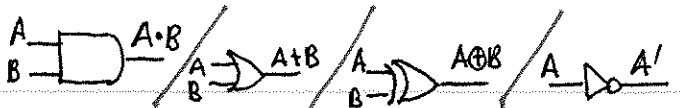
Regular algebra simplification:

$$xy + 3x + xy^2 + 3xy$$
$$= x(y+3+y^2+3y)$$
$$= x(y^2+4y+3)$$
$$= x(y+1)(y+3)$$

Boolean algebra

$$yx + yx' + xyx'$$
$$= y(x+x') + (xx')y$$
$$= y\cdot 1 + 0\cdot y$$
$$= y + 0$$
$$= y$$

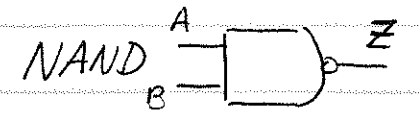
Gates

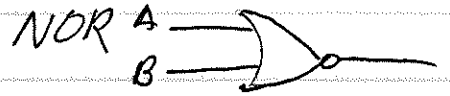


A	B	A · B (AND)	A + B OR	A ⊕ B XOR	A' NOT
0	0	0	0	0	1
0	1	0	1	1	1
1	0	0	1	1	0
1	1	1	1	0	0

0 — false

1 — true





Boolean algebra Laws

$A \cdot 1 = A$ $\quad A \cdot 0 = 0$

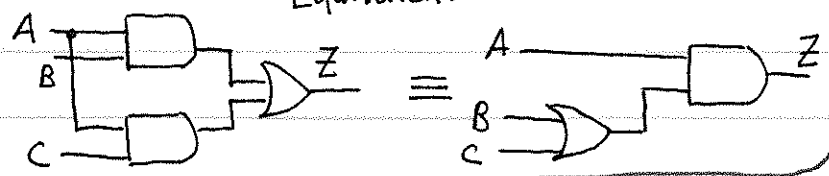
$A + 1 = 1$ $\quad A + 0 = A$

$A \cdot A' = 0$

$A + A' = 1$

$A \cdot B + A \cdot C = A \cdot (B + C)$

Equivalent



DeMorgan's Law

$(A + B)' = A' \cdot B'$ ①

$(A \cdot B)' = A' + B'$ ②

Verify ①

A	B	A + B	(A + B)'	A'	B'	A' · B'
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

The same