Number Systems

Base: The number of digits used to represent numbers

Binary: 0, 1 base = 2 e.g. 10110

Decimal: 0, 1, 2, ⋯, 9 base = 10 e.g. 254

Octal: 0, 1, 2, ⋯, 7 base = 8 e.g. 76^{o}

Hexdecimal: 0, 1, 2, ⋯, 9, A, B, C, D, E, F base = 16

e.g. AB3

positional Notation:

Decimal number: $254 = 2\times10^{2} + 5\times10^{1} \times 4\times10^{1}$

$= 200 + 50 + 4$

Binary number: $10110 = 1\times2^{4} + 0\times2^{3} + 1\times2^{2} + 1\times2^{1} + 0\times2^{0}$

$= 16 + 0 + 4 + 2 + 0$

$= 22$ (decimal equivalent)

In general: $\boxed{d_i \cdot base^{n_i} + d_{i-1} \cdot base^{n_{i-1}} + \cdots + d_1 \cdot base^{1} + d_0\, base^{0}}$

for any number with base

$n_i\ n_{i-1} \cdots 1\ 0$ ← weights

$d_i\ d_{i-1} \cdots d_1\ d_0$ ← digits

Coversion

Binary → decimal use the preceding formula and example.

decimal → binary

Divide the decimal number by 2 and the remainder will be the digit at 1's position. Then divid the quotient by 2, the remainder will be the digit at 2's position. Continue the process until the quotient is less than base

e.g. Convert 66 to binary number

33	16	8	4	2	1	
$2\overline{)66}$	$2\overline{)33}$	$2\overline{)16}$	$2\overline{)8}$	$2\overline{)4}$	$2\overline{)2}$	1
6	32	16	8	4	2	
6	1	0	0	0	0	
6						
0						

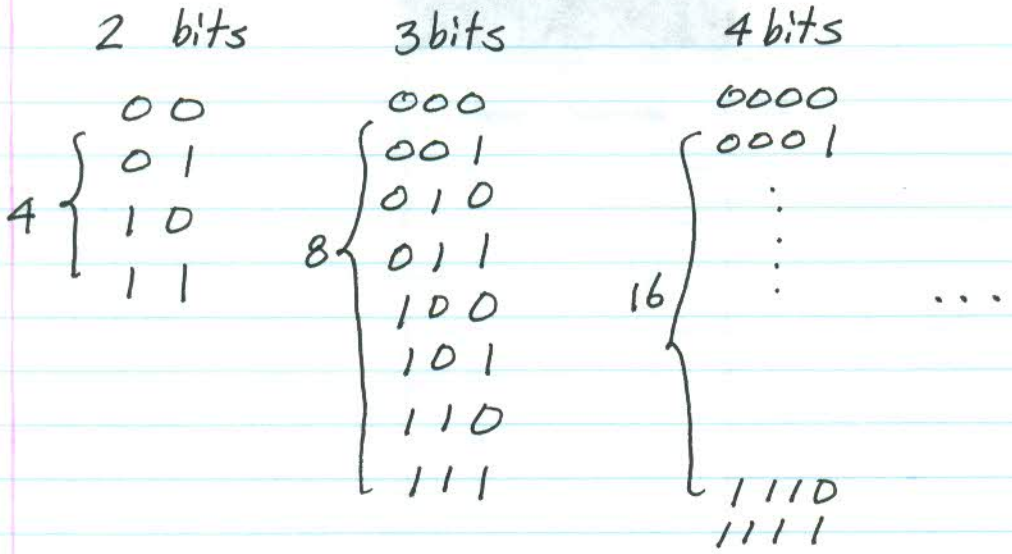
1000010

Number Systems

Number System	Digits	Base
Decimal	0, 1, 2, ··· 9	10
Binary	0, 1	2
Hexadecimal	0, 1, ··· 9, A, B, C, D, E, F (10, 11, 12, 13, 14, 15)	16
Octal	0, 1, ···, 7	8

$2^0 = 1$
$2^1 = 2$
$2^2 = 4$
$2^3 = 8$
$2^4 = 16$
$2^5 = 32$
$2^6 = 64$
$2^7 = 128$
$2^8 = 256$
$2^9 = 512$
$2^{10} = 1024 = 1K$ (Kilo)
$2^{20} = 1M$ (Mega)
$2^{30} = 1G$ (Giga)
$2^{40} = 1T$ (Tera)

Hex Digit		Binary
0	⟺	0000
1	⟺	0001
2		0010
3		0011
4		0100
⋮		⋮
9		1001
A (10)		1010
B (11)		1011
C (12)		1100
D (13)		1101
E (14)		1110
F (15)	⟺	1111



# of binary numbers $= 2^N$

(N is the number of bits)

e.g. $2^3 = 8$

$2^4 = 16$

Convert Hex → binary → Octal

A02 → 1010 0000 0010 → 5002$_8$

5 0 0 2

Binary → Hex (groups of 4bits)

1101 1100 → 0110 1100 → 6C

6 C

Binary → Octal

1101100 → 154$_8$

001 101 100

1 5 4

Addition & Substraction

```
  1   carry
  245
+  36
-----
  281
```

```
   ↘   borrow
  245
-  36
-----
  209
```

Binary numbers

```
   1
  1001   (9)
+ 0011   (3)
--------
  1100   (12)
```

```
    1 2
  1100   (12)
- 0011   (3)
--------
  1001   (9)
```